Brightness during a solar eclipse

David W. Hughes

A simple formula is developed that shows how the brightness of an observer's surroundings changes during a total solar eclipse. If it is cloudy it is clear that many unexpected partial eclipses go unnoticed.

Introduction

I have just lost my 'eclipse virginity'! The total solar eclipse of Wednesday, 1999 August 11, was the first total solar eclipse that I have ever seen. I had decided to stay in England, my feeling of nostalgia being enhanced by thoughts of two of my astronomical heroes, Edmond Halley[1] (the second Astronomer Royal) and Frank Watson Dyson[2] (the ninth) viewing 'their' English eclipses in 1715 and 1927. The fact that BT had invited me to their Earth Station Goonhilly Visitors' Centre on The Lizard in Cornwall to help explain the astronomical goings-on to assembled guests, had also convinced me to stay put in the land of my birth, and not forsake it for less cloudy points further east along the path of totality.

I was extremely lucky. Let me quote newspaper reporter Noel Perry's piece from Thursday August 12:[3]

> 'Like a miracle – that was the only possible description for the brief, but glorious appearance of the total eclipse of the sun on The Lizard yesterday.
>
> Less than five minutes before the appointed time, a cheer from the 200 or so guests at Goonhilly Earth Satellite Station signified that for the only time in the mid to late morning, the sun was visible. Until that time, all but the most optimistic had resigned themselves to the fact that they would not be seeing the eclipse. But there it was, the scimitar shaped sliver of sun, almost fully covered by the moon.
>
> Crowds continued to watch the sky and, particularly, the heavy clouds that still threatened to spoil the day Cornwall has been eagerly anticipating. Then, with a gasp from the crowds, there was totality, the much-vaunted Baily's Beads and the corona, as the eclipse was complete. It was such a surprise, in fact, that many failed to notice the other aspects of an eclipse – the drop in temperature or animals and birds becoming quiet. Then it was all over, just a couple of minutes later. And shortly after that, back came the clouds.
>
> Even the hard-bitten media were more than impressed, whooping and cheering with as much enthusiasm as the public.'

What specifically amazed me at the time was the darkness, or to be more precise, the lack of it. After reading widely I had geared myself up to expect day to be turned into night. The Meteorological Office (04:27 am, Mon 9 Aug) had given Cornwall a 15% chance of a good view of the eclipse. I was expecting complete cloud cover and dusk–night–dawn to be concertinaed into a one hour period. It wasn't like that.

At Goonhilly Downs, first, second, third and fourth contact took place at 09.57, 11.11, 11.13 and 12.32 BST respectively. By 10.40 I was beginning to panic. The sky was completely covered with clouds, and it was dismal, but no darker than one would expect on any other typical English cloudy day. The thought that I had completely miscalculated the time of the eclipse rushed through my mind. Dashing into the auditorium of the Visitors' Centre I was greatly cheered to see the large-screen television transmission showing that the Sun was already about 55% covered by the Moon. Why were my eyes so poor at detecting the diminution of the sunlight? Why was the decrease in general brightness so unnoticeable?

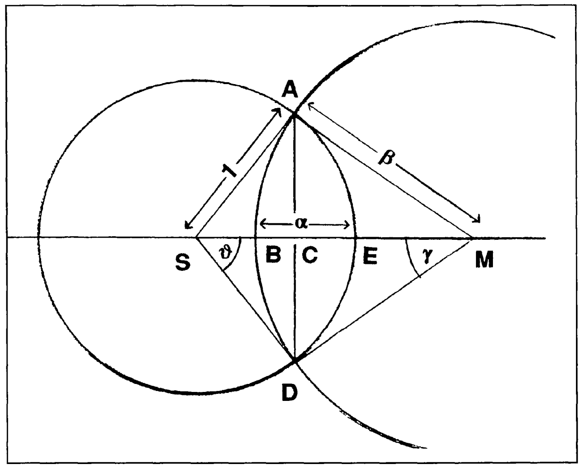

Figure 1. The eclipse has started and the Moon (centre M) has encroached onto the Sun (centre S) by a distance α. The angular diameter of the Sun is taken to be 1, and on 1999 August 11, the angular diameter of the Moon was β, where $\beta = 1.01436$. In the figure the difference between the two diameters has been exaggerated.

Brightness change during an eclipse

Assume for simplicity that the radius of the solar disk, as seen in the sky, is unity. Total eclipses occur when, among other things, the angular radius of the Moon is larger than that of the Sun. (If it were smaller the eclipse would be annular). Let the ratio between the radius of the Moon's disk and the Sun's disk (as seen from Earth) be β. [Around 10.12 UT on 1999 August 11, *The Astronomical Almanac, 1999* gave the data that the Sun was 151,621,000km away from Earth and the disk subtended 0°.525991, whereas the Moon was 373,270km away and subtended 0°.533545, so on this occasion $\beta = 1.01436$.]

Figure 1 shows the situation when the Moon has started to move across the face of the Sun. It has moved a distance α. Remembering that the radius of the circular solar disk has been taken to be 1.00, it can be seen that α is 0.00 at first contact and 2.00 at second contact. The value of α is directly proportional to the time that has elapsed since first contact. For the eclipse of 1999 August 11, the time between first and second contact for observers in Cornwall was 64 minutes.

We are now going to calculate the area of the lens ABDE in Figure 1, this being equivalent to the area of the eclipsed Sun. By using the cosine rule in triangle SDM it can be seen that the angle Θ in Figure 1 is given by

$$\cos\Theta = (1 + (1 + \beta - \alpha)^2 - \beta^2) / 2\,(1 + \beta - \alpha) \qquad [1]$$

The area of the sector ASDE is $2\pi\Theta / 360$.
The area of the triangle ASD is $\sin\Theta\cos\Theta$.
The area of the segment ACDE is thus
$(2\pi\Theta / 360) - \sin\Theta\cos\Theta$.
By a similar argument, the area of the segment ABDC is $(2\pi\beta^2\gamma / 360) - \beta^2 \sin\gamma\cos\gamma$. Comparing the length of the side CD in the two triangles SCD and MCD indicates that

$$\sin\Theta = \beta\sin\gamma \qquad [2]$$

The area of the lens ABDE is obtained by adding together the areas of the two segments ACDE and ABDC.

The uneclipsed area, A, of the Sun's disk shown in Figure 1, is given by

$$A = \pi - (2\pi\Theta / 360) + \sin\Theta\cos\Theta - (2\pi\beta^2\gamma / 360) + \beta^2\sin\gamma\cos\gamma \qquad [3]$$

Normalising this expression (i.e. making the area of the uneclipsed solar disk equal to 1.00), and converting γ values into Θ values using equation [2], gives the normalised visible bright area, A_N, of the solar disk seen during an eclipse as

$$A_N = 1 - (\Theta / 180) + \pi^{-1}\sin\Theta\cos\Theta - (\beta^2 / 180)\,[\sin^{-1}(\sin\Theta / \beta)] + \beta\pi^{-1}\sin\Theta\,(1 - [\sin\Theta / \beta\,]^2)^{0.5} \qquad [4]$$

The variation of A_N as a function of α (and thus the time between first and second contact) can be calculated using equations 1, 2 and 4. The results are shown in Figure 2, where the percentage of the solar disk that is obscured is

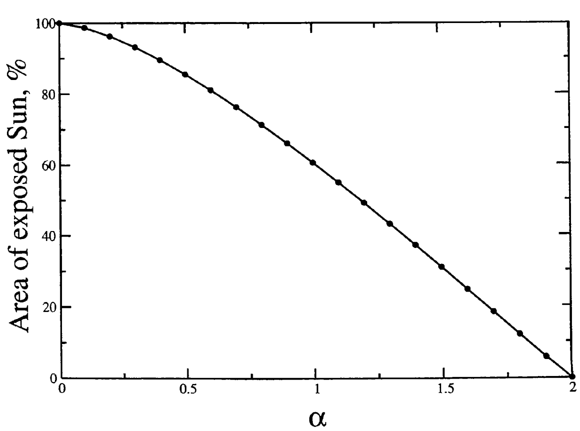

Figure 2. The graph shows how the percentage of the solar disk that is visible during a solar eclipse varies as a function of α. First contact occurs when $\alpha = 0.00$, and second contact when $\alpha = 2.00$. For the recent Cornish solar eclipse the time between these two contacts was 64 minutes.

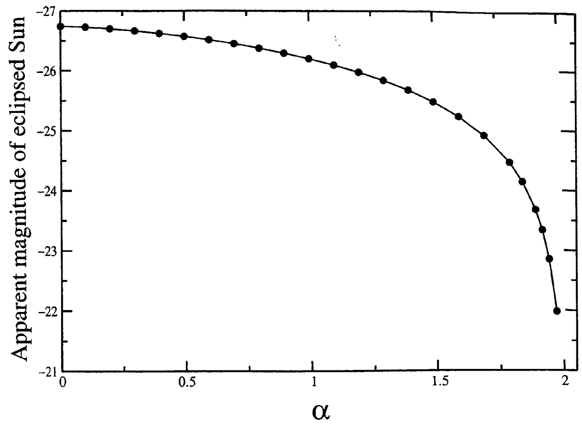

Figure 3. The apparent magnitude of the uneclipsed Sun is –26.74. The graph shows how the apparent magnitude varies during the eclipse. The tick marks on the abscissa occur every 0.25 in α, i.e. every 8 minutes.

plotted as a function of α. To convert from α to time (in minutes) one simply multiplies by 32.

Astronomers are used to working in terms of apparent magnitudes. This system is over two thousand years old and dragooned the visible naked-eye stars into six groups, those of first magnitude being the brightest and those of sixth magnitude the faintest that could be detected. The scale is actually logarithmic and it echoes the logarithmic nature of the response of the human eye to differences in brightness. A star of first magnitude is a hundred times brighter than one of sixth magnitude, and a first magnitude star is $100^{1/5}$ (i.e. 2.52) times brighter than one of second magnitude. On this scale the apparent visual magnitude of the uneclipsed Sun is –26.74.

Let the brightness of the uneclipsed and partially eclipsed Sun be b_O and b_{EO} respectively (brightness is the amount of energy an observer receives from a source per unit time and per unit area of detector). In what follows we assume that the Sun's disk is uniformly bright and is not affected by limb darkening. Here the brightness of the partially eclipsed Sun is proportional to the area of the disk that is visible, i.e.

$$b_{EO} / b_O = A_N.$$

Let the apparent visual magnitude of the partially eclipsed Sun be m_{EO}. From the relationship between apparent magnitude and brightness one then has

$$\log^{10}(b_{EO} / b_O) = \log_{10} A_N = 0.4\,(-26.74 - m_{EO}),$$

$$\text{i.e.}\quad m_{EO} = -26.74 - 2.5\log_{10} A_N \qquad [5]$$

where A_N is given by equation [4].

The variation of m_{EO} as a function of α (and thus the time between first and second contact) can be calculated using equations 1, 2, 4 and 5. The results are shown in Figures 3 and 4. Notice that the change in magnitude is quite slow until the last few minutes before the onset of totality at second contact.

Conclusions

Think of taking two photographs, around noon, one on a bright sunny day, and the other on a cloudy day. At a fixed

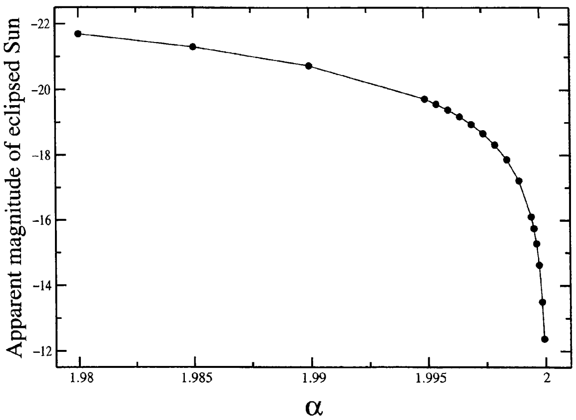

Figure 4. Totality starts when α = 2.00. The apparent magnitude of the eclipsed Sun changes drastically in the half minute before totality. For the Cornish eclipse, α = 1.98, 1.985, 1.99 and 1.995 are respectively 38.4, 28.8, 19.2, and 9.6 seconds before the onset of totality.

lens aperture the first could require an exposure of 1/1000 of a second whereas the latter might need an exposure of 1/60 of a second. Under both lighting conditions, humans, with their variable eye pupil diameters and logarithmic retinal responses, have no difficulty seeing things and moving about, even though the brightness of the scene has changed by a factor of nearly 17, and the magnitude of the light source by about 3.0.

I think a similar thing happens during the progression of an eclipse. If we are just looking at the ground and the surroundings, the apparent visual magnitude of the Sun has to decrease by about 3.0 before we appreciate that the light level has changed significantly. We thus sense that a significant decrease in the brightness of our surroundings has occurred only about three minutes before the onset of totality.

In Brighton, Bristol, London, Birmingham, Manchester, Belfast, Newcastle and Edinburgh the eclipse magnitude at maximum on 1999 August 11 was 98.6, 97.3, 96.8, 94.3, 91.6, 89.1, 87.5 and 85.2% respectively. (These percentages indicate how much of the solar disc was covered by the Moon). At maximum the apparent magnitude of the Sun would have been (from equation 5) –22.11, –22.82, –23.00, –23.63, –24.05, –24.33, –24.48, and –24.67 respectively. If a three-magnitude change is required for 'noticeability' then more than 93.7% of the solar disk has to be covered by the Moon for an unexpected partial eclipse to be noticeable if one is solely relying on brightness changes in one's surroundings.

The theoretical equations given above can be compared with experimental observations. A typical result is given in the book *The Science of Daylight*.[4] Also (staying in England) the staff of the Photometry Department of the National Physical Laboratory made careful measurements of the variation of illumination during the 1927 June 29 English eclipse using a Macbeth illuminometer.[5] These researchers compared the illumination provided by comparison lamps (powered by lead storage batteries) with the luminous flux that entered a circular horizontal hole in the top of a spherical white-coated diffusing sphere. Neutral filters were incorporated into the system and the accuracy of individual readings was about 5%.

The first reliable measurements of the change in the level of natural daylight during a total eclipse were made by Abney and Thorpe[6] in 1886.

Address: Department of Physics and Astronomy, The University, Sheffield, S3 7RH.

References

1 Edmond Halley, *Phil. Trans. Roy Soc. Lond.*, **29**, 245–262 (1715); **29**, 314–316 (1715); **32**, 197 (1722)
2 Margaret Wilson, *Ninth Astronomer Royal: The Life of Frank Watson Dyson*, Heffer & Sons, Cambridge, 1951, p. 224–227. Quoting from page 225: 'It turned 6.0 and the now almost eclipsed sun was behind clouds again. Totality was almost due. Then, as Lady Dyson described the scene to her family afterwards: 'it was like the opening of a play. The clouds rolled slowly back like the great curtains, and there was the stage set.' In the centre of the stage appeared the sun, almost – and then quite – blotted out by the moon's disc, and around it was a dazzling circle of fiery light, from which red and yellow flames shot out. For twenty-three seconds, in a weird kind of twilight, the observers changed their plates, and the general public gazed in astonished awe. Ruth was struck with the movement of the birds, which, after chirruping cheerfully to salute the dawn, now circled around uneasily, not knowing whether it was night or day. Sheep, too, huddled close together against the walls. Apart from the eerie light, the air struck chill. There was actually a drop in temperature of about ten degrees. As the moon slowly slipped away and a white sickle of the sun reappeared, there was a burst of cheering. Another total eclipse was over.'
3 Noel Perry, *West Briton & Royal Cornwall Gazette*, Thursday 1999 August 12, p.1
4 John W. T. Walsh, *The Science of Daylight*, Macdonald, London, 1961, p. 48
5 *The Illumination Engineer*, July 1928, p. 198
6 W. de W. Abney & Thorpe, *Phil. Trans. Roy. Soc. Lond.*, **180**, 363 (1889)

Received 1999 September 4; accepted 1999 October 27